

Binomial Expansions

The binomial theorem is used to expand brackets. It states:

$$(x + y)^n = \sum_{r=0}^n {}^nC_r x^{n-r} y^r$$

Example: Expand $(x + y)^6$

$$\begin{aligned}(x + y)^6 &= {}^6C_0 x^{6-0} y^0 + {}^6C_1 x^{6-1} y^1 + \dots + {}^6C_6 x^{6-6} y^6 \\&= 1 \cdot x^6 \cdot 1 + 6 \cdot x^5 \cdot y + \dots + 1 \cdot 1 \cdot y^6 \\&= x^6 + 6x^5y + \dots + y^6\end{aligned}$$

The general term in $(x + y)^n$ is:

$$T_{r+1} = {}^nC_r x^{n-r} y^r$$

The constant term is the one with x^0 (i.e. independent of x)

Questions:

1. Find the general term for $(2x - 5y)^{11}$

$$T_{r+1} = {}^{11}C_r (2x)^{11-r} (-5y)^r$$

2. State the 4th term in the expansion $(3x + 2)^8$

$$\text{General term: } T_{r+1} = {}^8C_r (3x)^{8-r} (2)^r$$

$$\begin{aligned}4^{\text{th}} \text{ term is when } r = 3: T_4 &= {}^8C_3 (3x)^{8-3} (2)^3 \\&= {}^8C_3 3^5 x^5 2^3 \\&= 108864x^5\end{aligned}$$

3. What is the constant term in the expansion of $(x^2 + \frac{3}{x})^6$?

$$\begin{aligned}T_{r+1} &= {}^6C_r (x^2)^{6-r} \left(\frac{3}{x}\right)^r = {}^6C_r x^{12-2r} \frac{3^r}{x^r} = {}^6C_r x^{12-2r} 3^r x^{-r} \\&= {}^6C_r x^{12-3r} 3^r \quad (\text{adding powers of } x)\end{aligned}$$

The constant term occurs when the power of x is 0, i.e. when $12 - 3r = 0$.
Therefore the constant term occurs when $r = 4$, the 5th term.

$$T_{4+1} = T_5 = {}^6C_4 x^0 3^4 = 1215.$$

Delta: Ex 27.6 pages 254 – 255, Q 1 – 3, 7. Extension: everything else.